AQA Mechanics

Topic Questions from Papers

Differential Equations

Answers

1	$1600\frac{dv}{dt} = -40v$	M1 A1		Applying Newton's second law with $40v$ and $\frac{dv}{dt}$. Correct equation
	$\int \frac{1}{v} dv = \int -\frac{1}{40} dt$	dM1		Separating variables
	$\ln v = -\frac{t}{40} + c$	dM1		integrating to get ln$v$ term.
	$v = Ae^{-\frac{t}{40}}$	A1		Correct integral with or without c
	$t = 0, v = 20 \Rightarrow c = 20$	dM1		Finding constant
	$v = 20e^{-\frac{t}{40}}$	A1	7	Correct final result
	Total		**7**	

(Q5, Jan 2006)

2 (a)	$20\frac{\mathrm{d}v}{\mathrm{d}t} = -10\sqrt{v}$	M1		applying Newton's second law with $\frac{\mathrm{d}v}{\mathrm{d}t}$
	$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{\sqrt{v}}{2}$	A1 dM1		correct differential equation separating variables
	$\int \frac{1}{\sqrt{v}}\mathrm{d}v = \int -\frac{1}{2}\mathrm{d}t$ AG			
	$2\sqrt{v} = -\frac{t}{2} + c$	dM1 A1		integrating correct integrals
	$t = 0, v = 25 \Rightarrow c = 10$	dM1		finding the constant of integration
	$v = \left(\frac{20-t}{4}\right)^2$	A1	7	correct final result from correct working
(b)	$t = 20$	B1	1	correct time
	Total		**8**	

(Q6, June 2006)

3 (a)	Max speed $\equiv$ zero acceleration used	M1		Implied
	$\frac{72000}{60}$	M1		
	$\frac{72000}{60} = k \times 60$			
	$k = 20$	A1	3	
(b)(i)	$20v = -500\frac{dv}{dt}$	M1		see $\frac{dv}{dt}$, $\pm$
	$\frac{dv}{dt} = -\frac{v}{25}$	A1	2	
(ii)	$25\int \frac{dv}{v} = -\int dt$	M1 A1		M1 separating variables
				Alternative
	$[25\ln v]_{20}^{10} = -[t]_0^t$	A1		$25 \ln v = -t\ (+ c)$ A1
	$25\ln 10 - 25\ln 20 = -t$	m1 A1		$t = 0$, $v = 20$, $c = 25\ln 20$ m1 $t = t$, $v = 10$, $25\ln 10 = -t + 25\ln 20$ A1
	$t = 25\ln 2$ or 17.3 or $-25\ln\frac{1}{2}$	A1	6	$t = 25\ln 2$ or 17.3 A1
	Total		**11**	

(Q7, Jan 2007)

4 (a)	Using $F = ma$:			
	$-\lambda mv = ma = m\frac{dv}{dt}$	M1		Condone no '–'
	$\therefore \frac{dv}{dt} = -\lambda v$	A1	2	AG Note: no use of $m \Rightarrow$ no marks in (a)
(b)	$\int \frac{dv}{v} = -\lambda \int dt$	M1		
	$\ln v = -\lambda t + c$ $v = Ce^{-\lambda t}$	A1		Needs '+ c'
	When $t = 0$, $v = U \Rightarrow C = U$	M1		Needs correct working
	$v = U e^{-\lambda t}$	A1	4	AG
	Total		**6**	

(Q7, June 2007)

5 (a)	Power of engine is 8kW			
	$\therefore$ Force exerted by engine $= \frac{8000}{v}$	M1A1		M1 for Power = Fv
	Using $F = ma$:	m1		
	$\frac{8000}{v} - kv^2 = 600\frac{\mathrm{d}v}{\mathrm{d}t}$			
	$600\frac{\mathrm{d}v}{\mathrm{d}t} - \frac{8000}{v} + kv^2 = 0$	A1	4	AG
(b)(i)	When engine is turned off, power is zero:			
	$-kv^2 = 600\frac{\mathrm{d}v}{\mathrm{d}t}$	B1	1	AG
(ii)	$\int 600\frac{\mathrm{d}v}{v^2} = -\int k\,\mathrm{d}t$	M1		
	$-\frac{600}{v} = -kt + c$	A1		Need '+ c'
	When $t = 0$, $v = 20$:			
	$\therefore c = -\frac{600}{20} = -30$	A1		
	$\therefore \frac{600}{v} = kt + 30$			
	When $v = 10$, $kt = 30$:	M1		
	$\therefore t = \frac{30}{k}$	A1	5	$-\frac{30}{k}$ SC3
	Total		**10**	

(Q8, Jan 2008)

6 (a)	Using $F = ma$			
	$-0.05mv = m\frac{\mathrm{d}v}{\mathrm{d}t}$			
	$\therefore \frac{\mathrm{d}v}{\mathrm{d}t} = -0.05v$	B1	1	Need to see m terms
(b)	$\int \frac{\mathrm{d}v}{v} = -\int 0.05\,\mathrm{d}t$	B1		
	$\ln v = -0.05t + c$	M1		Need first 2 terms
	$v = Ce^{-0.05t}$			
	When $t = 0$, $v = 20$,			
	$\therefore C = 20$	M1		} fully correct solutions
	$v = 20e^{-0.05t}$	A1	4	
(c)	When $v = 10$, $10 = 20e^{-0.05t}$	M1		
	$e^{0.05t} = 2$	A1		
	$\therefore t = \frac{1}{0.05}\ln 2$			
	$= 13.9$	A1	3	Accept 20 ln 2
	Total		**8**	

(Q6, June 2008)

7 (a)	Using $F = ma$: $-0.08v^2 = 0.05a$ $\therefore \frac{dv}{dt} = -1.6v^2$	B1 B1	2	AG; condone sign error in first B1
(b)	$\int \frac{dv}{v^2} = -1.6\int dt$ $-\frac{1}{v} = -1.6t\ (+c)$ When $t = 0$, $v = 3 \Rightarrow\ c = -\frac{1}{3}$ $\frac{1}{v} = \frac{1}{3} + 1.6t$ * $\frac{1}{v} = \frac{1}{3} + \frac{8}{5}t$ $\frac{1}{v} = \frac{5+24t}{15}$ $v = \frac{15}{5+24t}$	M1 A1 M1 A1 A1	5	Condone $-\frac{1}{v} = -1.6t + c \Rightarrow \frac{1}{v} = 1.6t + c$ AG; all working lines correct from *
	Total		**7**	

(Q8, Jan 2009)

8 (a)	Using $F = ma$: $-\lambda m v^{\frac{3}{2}} = m\frac{dv}{dt}$ $\therefore \frac{dv}{dt} = -\lambda v^{\frac{3}{2}}$	M1 A1	2	AG
(b)	$\int \frac{dv}{v^{\frac{3}{2}}} = -\lambda \int dt$ $-\frac{2}{v^{\frac{1}{2}}} = -\lambda t + c$ When $t = 0$, $v = 9 \Rightarrow c = -\frac{2}{3}$ $\frac{2}{\sqrt{v}} = \lambda t + \frac{2}{3}$ $\frac{\sqrt{v}}{2} = \frac{1}{\lambda t + \frac{2}{3}}$ $v = \left(\frac{6}{2+3\lambda t}\right)^2$ $v = \frac{36}{(2+3\lambda t)^2}$	M1 A1 M1 A1 A1 m1 A1	7	 Condone no '+c' Dep. on correct integration (accept sign or '$\frac{1}{2}$' error) Needs correct algebra AG
(c)	When $v = 4$, $\frac{2}{\sqrt{v}} = \lambda t + \frac{2}{3} \Rightarrow 1 = \lambda t + \frac{2}{3}$ $t = \frac{1}{3\lambda}$	 M1A1 A1	3	**or** $\frac{36}{(2+3\lambda t)^2} = 4$ M1 $(2+3\lambda t)^2 = 9$ A1 $t = \frac{1}{3\lambda}$ A1 needs statement why $2+3\lambda t \neq -3$
	Total		**12**	

(Q8, June 2009)

9 (a)	Using $F = ma$, $-0.2mv^{\frac{1}{2}} = m\frac{dv}{dt}$ $\therefore \frac{dv}{dt} = -0.2v^{\frac{1}{2}}$	B1	1	AG Must see equ'n containing m
(b)	$\int \frac{dv}{v^{\frac{1}{2}}} = -\int 0.2\ dt$	M1		
	$2v^{\frac{1}{2}} = -0.2t + c$	A1m1		m1 for + c
	When $t=0$, $v=16 \therefore C=8$	A1		
	$2v^{\frac{1}{2}} = -0.2t + 8$ $v = (4-0.1t)^2$	A1	5	AG
(c)	When $v=1$, $1=(4-0.1t)^2$ $4 - 0.1t = \pm 1$	M1		
	$t=30$ or 50	A1		$\left[\text{if use } 2v^{\frac{1}{2}} = 8 - 0.2t \text{ no need to see } 50\right]$
	$t=30$	A1	3	$t \neq 50$ as ball stops when $t = 40$
(d)	Integrating $v=(4-0.1t)^2$: $v = 16 - 0.8t + 0.01t^2$ $x = 16t - 0.4t^2 + \frac{0.01}{3}t^3 + d$	M1		M1 for first 3 terms or $-\frac{10}{3}\ (4-01t)^3$
	When $t=0$, $x=0 \Rightarrow d=0$	A1		
	$x = 16t - 0.4t^2 + \frac{0.01}{3}t^3$ When speed is 1ms^{-1}, $t = 30$ $x = 480 - 360 + 90$	m1		dep on M1 above
	$= 210$	A1	4	[No 'd', 3 marks only]
	Total		**13**	

(Q5, Jan 2010)

10	$\frac{dv}{dt} = -\frac{\lambda}{v^{\frac{1}{4}}}$	M1		
	$\int v^{\frac{1}{4}} dv = -\int \lambda \, dt$	m1		Condone one of $v^{-\frac{1}{4}}$, $+\int \lambda dt$, $\frac{1}{\lambda}$
	$\frac{4}{5} v^{\frac{5}{4}} = -\lambda t + c$	A1A1 m1		m1 for + c
	$t = 0,\ v = u\ \therefore\ c = \frac{4}{5} u^{\frac{5}{4}}$	A1		
	$\therefore v^{\frac{5}{4}} = u^{\frac{5}{4}} - \frac{5}{4}\lambda t$			
	$v = \left(u^{\frac{5}{4}} - \frac{5}{4}\lambda t \right)^{\frac{4}{5}}$	A1	7	
	Total		**7**	

(Q5, June 2010)

11 (a)(i)	$F = 65g - 260v$ $= 65(9.8 - 4v)$	B1	1	Accept $260v - 65g$ AG must see $65g$ or 260
(ii)	Using $F = ma$			
	$65 \frac{dv}{dt} = 65(9.8 - 4v)$	M1		Need to see terms in m (condone − sign)
	$\frac{dv}{dt} = -4(v - 2.45)$	A1	2	AG
(b)	$\frac{1}{v - 2.45} \frac{dv}{dt} = -4$	B1		
	$\int \frac{1}{v - 2.45} dv = -\int 4 \, dt$			
	$\ln(v - 2.45) = -4t + c$	M1 A1		M1 log side correct $-4t + c$
	$v - 2.45 = Ce^{-4t}$ $t = 0,\ v = 19.6$ $\therefore\ C = 17.15$ or $e^{2.84}$	A1		Or $c = \ln 17.15$ or 2.84
	$\therefore v = 2.45 + 17.15e^{-4t}$ $\quad 2.45 + 17.2e^{-4t}$	A1	5	
	Total		**8**	

(Q8, Jan 2011)

12 (a)	Using $F = ma$ $-2mv^{\frac{5}{4}} = m\frac{\mathrm{d}v}{\mathrm{d}t}$ $\therefore \frac{\mathrm{d}v}{\mathrm{d}t} = -2v^{\frac{5}{4}}$ **AG**	B1	1	B1: Must see $-2mv^{\frac{5}{4}} = m\frac{\mathrm{d}v}{\mathrm{d}t}$ or $-2mv^{\frac{5}{4}} = ma$ and correct final answer.
(b)	$\int \frac{\mathrm{d}v}{v^{\frac{5}{4}}} = -2\int \mathrm{d}t$	M1		M1: Two integrals with one in the form $\int f(v)dv$ where $f(v) = v^{\pm\frac{5}{4}}$ or $v^{\pm\frac{4}{5}}$. The other integral must not contain v terms.
	$-\frac{4}{v^{\frac{1}{4}}} = -2t + c$	A1		A1: Correct expression. Condone lack of + c for this A1, but no subsequent marks if no c.
	When $t = 0$, $v = 16 \Rightarrow c = -2$	dM1 A1		dM1: Using $t = 0$ and $v = 16$ to find c. A1: Obtaining $c = -2$.
	$-\frac{4}{v^{\frac{1}{4}}} = -2t - 2$ $v^{\frac{1}{4}} = \frac{2}{1+t}$ $v = \left(\frac{2}{t+1}\right)^4$ **AG**	A1	5	A1: Correct final answer. Must see $v^{\frac{1}{4}} = \frac{2}{1+t}$ or $v^{-\frac{1}{4}} = \frac{1+t}{2}$ or $\frac{1}{v^{\frac{1}{4}}} = \frac{1+t}{2}$ Or if they obtain $v = \left(\frac{2}{t+c}\right)^4$ $v = 16, t = 0 \Rightarrow 16^{\frac{1}{4}} = \frac{2}{c}$, condone $c = 1$ (no other root considered)
	Total		**6**	

(Q6, June 2011)

13 (a)	using $F = ma$			
	$0.4\frac{dv}{dt} = 2 - 4v$	M1		
	$\frac{dv}{dt} = -10(v - 0.5)$	A1	2	Needs line above
(b)	hence $\int \frac{1}{v-0.5} dv = -\int 10 dt$			
	$\ln(v - 0.5) = -10t + c$	M1A1 m1		M1 for any side integrated correctly m1 for + c (and M1 gained)
	$v - 0.5 = Ce^{-10t}$			
	$t = 0, v = 1$			
	$\therefore C = 0.5$	A1		
	$\therefore v = 0.5 + 0.5e^{-10t}$	A1	5	condone $v = 0.5 + e^{-10t-0.693}$
(c)	when $v = 0.55$, $0.55 = 0.5 + 0.5e^{-10t}$	M1		substitute 0.55 into C's (b), after finding c, possible numerical error
	$10 = e^{10t}$	A1		
	$t = \ln 10 \div 10$			
	$= 0.230$	A1	3	
	Total		**10**	

(Q6, Jan 2012)

14 (a)	Using $F = ma$:			
	$m\frac{dv}{dt} = 49 - 9.8v$ or $5g - 9.8v$	M1		Need to see $m\frac{dv}{dt}$ or $5\frac{dv}{dt}$ or $a = \frac{49-9.81}{5}$
	$\therefore \frac{dv}{dt} = -1.96(v - 5)$	A1	2	Must see m terms (not $a = \ldots$)
(b)	$\int \frac{dv}{v-5} = -1.96\int dt$	M1		And one side integrated
	$\ln(v - 5) = -1.96t + c$	A1A1		Need + c, A1 each side
	When $t = 0$, $v = 7 \Rightarrow c = \ln 2$	A1		OE
	$\ln\frac{v-5}{2} = -1.96t$			
	$\frac{v-5}{2} = e^{-1.96t}$			
	$v = 5 + 2e^{-1.96t}$	A1	5	CAO
	Total		**7**	

(Q7, June 2012)

15 **(a)**	Using F = ma:			
	$-4v^{\frac{1}{3}} = 12\frac{dv}{dt}$	B1		
	$\therefore \frac{dv}{dt} = -\frac{1}{3}v^{\frac{1}{3}}$			
	$-3\int \frac{dv}{v^{\frac{1}{3}}} = \int dt$	M1		condone $-$, 3 incorrect side
	$-3\times\frac{v^{\frac{2}{3}}}{\frac{2}{3}} = t + c$	A1		condone lack of + c
	$-\frac{9}{2}v^{\frac{2}{3}} = t + c$			
	When $t = 0$, $v = 8 \Rightarrow c = -18$	M1A1		
	$-\frac{9}{2}v^{\frac{2}{3}} = t - 18$			
	$v^{\frac{2}{3}} = 4 - \frac{2}{9}t$			
	$v = \left(4 - \frac{2}{9}t\right)^{\frac{3}{2}}$	A1	6	
(b)	Particle is at rest when $4 - \frac{2}{9}t = 0$			
	The value of t is 18	B1	1	
	Total		**7**	

(Q5, Jan 2013)

16 **(a)**	Using $F = ma$			
	$1600\frac{dv}{dt} = 4000 - 40v$	M1		
	$\frac{dv}{dt} = \frac{4000 - 40v}{1600}$			
	$\frac{dv}{dt} = \frac{100 - v}{40}$	A1	2	
(b)	$40\frac{dv}{100 - v} = dt$	B1		
	$40\int \frac{dv}{100 - v} = \int dt$	M1		
	$-40\ln(100 - v) = t + c$	A1		Condone lack of '+ c'
	When $t = 0$, $v = 0 \Rightarrow c = -40\ln 100$	M1A1		
	$-40\ln(100 - v) = t - 40\ln 100$			
	$t = 40\ln\frac{100}{100 - v}$			
	$e^{\frac{t}{40}} = \frac{100}{100 - v}$			
	$v = 100 - 100e^{-\frac{t}{40}}$ or $100(1 - e^{-\frac{t}{40}})$	A1	6	
	Total		**8**	

(Q6, June 2013)